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PROBABILISTIC MODEL OF A PARTICLE-ROUGH WALL COLLISION

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A statistical model of a collision of particles against a randomly rough surface is proposed. Closed expressions for the density functions of distribution of the coefficients of momentum regeneration are obtained. It is shown that, for small angles of incidence of the particles, the coefficient of regeneration of the normal component of the momentum on a rough surface can be greater than unity.

Disperse materials in piping systems can be pneumatically conveyed by a turbulent flow in two fundamentally different regimes: 1) the transportation of fine particles whose dynamic relaxation time is of the order of the integral time scale of turbulence (the disperse impurity of these particles is involved in the energy-consuming fluctuations of the carrying-gas velocity; the two-phase flow of fine particles in pipes behaves like a homogeneous system); 2) the transportation of coarse particles whose dynamic relaxation time significantly exceeds the characteristic lifetime of energy-consuming turbulent vortices (the chaotic motion of the impurity is attributed to interparticle collisions and collisions against the channel surface). It is noteworthy that conveying of coarse particles can be more effective owing to less effort expended for particle grinding.

The stable operation of pneumatic transport systems depends on the homogeneity of the concentration distribution of the disperse impurity in the cross section. The increase in the intensity of chaotic motion of the particles results in a more uniform concentration profile. For coarse particles, the velocity fluctuations are mainly due to their collisions against the walls. The random components of the particle velocity are attributed to the nonspherical shape of the particles and the roughness of the channel surface. When the particles interact with the wall, their rotation about the contact point is observed. The gravitational force and the loss of the axial velocity of the particles give rise to high-velocity slip of the phases. The differences in the gas and particle velocities and their rotation result in the emergence of the Magnus force, which causes the intense transverse movement of the particles in the channel. To predict the motion of coarse particles in channels, a model of the collision between the particles and a randomly rough surface is needed. In this paper, the collision between spherical particles and a rough wall is studied.

Matsumoto et al. [1, 2] modeled the rough wall by a sinusoidal surface with the use of the method of numerical calculation of random particle trajectories in Lagrange variables. The results of a combined theoretical and experimental investigation of collisions between the particles and a randomly rough surface were given by Sommerfeld in [3, 4]. The rough wall in [3, 4] is described by planar surfaces having random angles of slope relative to the channel axis. A particle-irregular surface collision was analyzed by direct numerical modeling of stochastic trajectories. However, in this problem the use of the method of direct numerical modeling, which requires considerable CPU time using supercomputers, is not justified and does not allow one to reveal the main parameters that control the process of collision between the particles and a random surface.

In this paper, we use the theory of stochastic processes [5] to derive analytical formulas for the probability density of distribution of the coefficients of regeneration of the particle momentum as a function of the physical properties of the particle and wall materials, the particle diameters, and the characteristic

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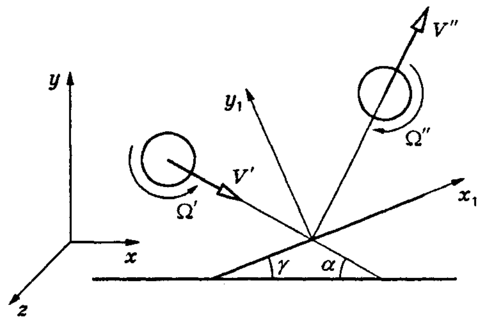

Fig. 1

dimension of the roughness. The formulas can be used to formulate the boundary conditions for the equations of turbulent motion of a disperse impurity with allowance for particle rotation in Euler variables [6, 7]. The predicted parameters of the particles reflected from a rough wall are compared with the experimental data of [3, 4].

1. Model of a Particle–Rough Wall Collision. We consider a surface with random sand roughness characterized by the height of random mounds and the distance between the mounds. We confine our analysis to a surface with mounds whose characteristic height is less than the particle diameter. If the particle diameter is smaller than the characteristic distance between random bulges on the surface, the parameters of the particles reflected from the surface depend on the microstructure of the surface. The particles whose diameter exceeds the characteristic distance between the mounds collide against several random bulges on the surface, which leads to "averaging" of the random structure. Moreover, the effective height of the random roughness elements for these particles is smaller compared to the case of fine particles.

The roughness element is modeled by a plane having a random angle of slope γ relative to the channel axis (Fig. 1). The angle of incidence α and the random angle of slope of the plane γ are reckoned from a smooth surface parallel to the channel wall. The components of the linear velocity $\boldsymbol{V}$ and the angular velocity of the particles rotating about their axes $\boldsymbol{\Omega}$ before collision are primed, and these parameters after collision are double-primed.

The velocity components in the coordinate system (x_1, y_1) of the inclined plane are related to the velocity components in the coordinate system (x, y) attached to the channel wall by the formulas

$$V'_x(\gamma) = V'_x \cos\gamma - V'_y \sin\gamma, \qquad V'_y(\gamma) = V'_y \cos\gamma + V'_x \sin\gamma. \tag{1.1}$$

Here V'_x, V'_y and $V'_x(\gamma)$, $V'_y(\gamma)$ are the components of the particle velocity before the collision in the coordinate systems (x, y) and (x_1, y_1), respectively.

After a collision against a random surface, the components of the velocity relative to the channel wall are expressed via the velocity components of the particle reflected from a random inclined surface:

$$V''_x = V''_x(\gamma)\cos\gamma - V''_y(\gamma)\sin\gamma, \qquad V''_y = V''_y(\gamma)\cos\gamma + V''_x(\gamma)\sin\gamma. \tag{1.2}$$

Here V''_x, V''_y and $V''_x(\gamma)$, $V''_y(\gamma)$ are the components of the particle velocity relative to the channel wall and the random plane, respectively, after the collision.

The particles collide only against random planes whose angle of slope satisfies the condition $\gamma \geqslant -\alpha$. Using the assumption of the sand roughness of the channel wall, we approximate the angle of slope of random planes by a normal distribution with zero mean value and variance Δ. The probability that a particle collides against a random surface depends on its angle of incidence. The angle of slope of the plane relative to the channel wall γ at which a collision between a particle and a random surface can occur is a random collision angle. Using the theory of random Brownian motion [5], we construct a function of conventional probability density of distribution of the collision angle

$$P(\gamma,\alpha)=\frac{1}{\sqrt{2\pi}\Delta}\Big[\mathrm{erf}\Big(\frac{\alpha}{\sqrt{2}\Delta}\Big)\Big]^{-1}\Big[\exp\Big(-\frac{\gamma^2}{2\Delta^2}\Big)-\exp\Big(-\frac{(2\alpha+\gamma)^2}{2\Delta^2}\Big)\Big],$$
$$P(\gamma,\alpha)=0 \quad \text{for} \quad \gamma\leqslant-\alpha. \tag{1.3}$$

Here $\mathrm{erf}\,(x)=\frac{2}{\sqrt{\pi}}\int\limits_0^x \exp\,(-t^2)\,dt$ is the standard probability integral. The density of the distribution function (1.3) satisfies the normalization condition $\int\limits_{-\infty}^{\infty} P(\gamma,\alpha)\,d\gamma=1$.

If the angles of incidence of the particles significantly exceed the variance of the random collision angle $\alpha\gg\Delta$, the function (1.3) becomes the standard normal distribution $P(\gamma,\alpha)=1/(\sqrt{2\pi}\Delta)\exp\,(-\gamma^2/(2\Delta^2))$.

For small angles of incidence $\alpha\ll\Delta$, the conventional probability (1.3) corresponds to the Rayleigh function of probability density [5] $P(\gamma,\alpha)=\gamma/\Delta^2\exp\,(-\gamma^2/(2\Delta^2))$.

The function of conventional probability (1.3) is used to determine the mean value and variance of the random collision angle

$$\langle\gamma\rangle=\alpha\,\frac{1-\mathrm{erf}(A)}{\mathrm{erf}(A)},\qquad A=\frac{\alpha}{\sqrt{2}\Delta}; \tag{1.4}$$

$$D(\gamma)=(\langle\gamma^2\rangle-\langle\gamma\rangle^2)^{1/2}. \tag{1.5}$$

The mean value of the squared random angle γ is

$$\langle\gamma^2\rangle=\Delta^2[\mathrm{erf}(A)]^{-1}\Big\{1+\frac{2}{\sqrt{\pi}}A\exp\,(-A^2)-4A^2[1-\mathrm{erf}(A)]\Big\}. \tag{1.6}$$

It follows from (1.4)–(1.6) that, for sufficiently large angles of incidence of the particles $\alpha\gg\Delta$, the mean value of the random angle $\langle\gamma\rangle\to0$, and its variance reaches the maximum value: $D(\gamma)=\Delta$. For small angles of incidence $\alpha\ll\Delta$, we obtain the maximum mean value of the angle and the minimum variance: $\langle\gamma\rangle\to\sqrt{\pi/2}\Delta$ and $D(\gamma)\to\Delta(2-\pi/2)^{1/2}$.

It follows from (1.1)–(1.3) that to determine the relation between the velocities of the incident and reflected particles, a model that describes the process of their collision against a planar surface is required.

2. Model of a Particle–Surface Collision. Effective Coefficients of Momentum Regeneration. We consider a particle–surface collision at comparatively small velocities, which are characteristic of the regime of pneumatic transport of a disperse impurity (of the order of a dozen meters per second). In this case, the plastic deformation of the particle and channel materials can be ignored.

The conversion of the linear and angular velocities due to collision against a plane is described within the framework of the model obtained from the momentum balance during collision [1, 8]. We distinguish between two regimes of collision: with particle slippage relative to the surface and without this slippage. The nonslip condition has the following form [1, 8]:

$$\Big|V_x'(\gamma)+\frac{d_p}{2}\,\Omega_z'(\gamma)\Big|\leqslant\frac{7}{2}\,\mu_0(1+e)|V_y'(\gamma)|. \tag{2.1}$$

Here d_p is the particle diameter, μ_0 is the coefficient of static friction, and e is the coefficient of regeneration of the momentum normal component.

After a slippage-free collision, the velocity components become

$$V_x''(\gamma)=\frac{5}{7}\,V_x'(\gamma)-\frac{1}{7}\,d_p\Omega_z'(\gamma),\quad V_y''(\gamma)=-eV_y'(\gamma),\quad \Omega_z''(\gamma)=-2\,\frac{V_x'(\gamma)}{d_p}. \tag{2.2}$$

In the case where the inequality (2.1) fails, the regime with slippage occurs, and the velocity components after collision take the form

$$V_x''(\gamma) = \frac{5}{7} V_x'(\gamma) + \mu_d \varepsilon_0 (1 + e) V_y'(\gamma), \qquad V_y''(\gamma) = -e V_y'(\gamma),$$
$$\Omega_z''(\gamma) = \Omega_z'(\gamma) + 5\mu_d (1 + e) \varepsilon_0 \frac{V_y'(\gamma)}{d_p}, \tag{2.3}$$

where μ_d is the coefficient of dynamic friction and ε_0 is an index that takes on the values ± 1 depending on the direction of the particle velocity relative to the wall:

$$\varepsilon_0 = \operatorname{sign}\Big(V_x'(\gamma) + \frac{d_p}{2} \Omega_z'(\gamma)\Big). \tag{2.4}$$

We determine the effective coefficients of momentum regeneration and conversion of the angular velocity of the particles relative to the channel wall:

$$k_t = \frac{V_x''}{V_x'}, \qquad k_n = \left|\frac{V_y''}{V_y'}\right|, \qquad k_\Omega = \frac{d_p \Omega_z''}{2V_x'}. \tag{2.5}$$

Using the transformations of the velocity components (1.1) and (1.2) and the relations between the parameters of the particle after collision against a random surface (2.1)–(2.4), we obtain closed expressions for the coefficients of momentum regeneration (2.5) relative to the channel wall.

In the absence of slippage [condition (2.1)], we have the relations

$$|\cos\gamma - z\sin\gamma + A_\Omega| \leqslant \frac{7}{2} \mu_0 (1 + e) |z\cos\gamma + \sin\gamma|,$$
$$k_t = \Big[\frac{5}{7}(\cos\gamma - z\sin\gamma) - \frac{2}{7} A_\Omega\Big] \cos\gamma - e(z\cos\gamma + \sin\gamma)\sin\gamma,$$
$$k_n = e\Big(\cos\gamma + \frac{\sin\gamma}{z}\Big)\cos\gamma + \Big[\frac{5}{7}\Big(\frac{\cos\gamma}{z} - \sin\gamma\Big) - \frac{2}{7} A_\Omega\Big] \sin\gamma, \tag{2.6}$$
$$k_\Omega = -\frac{5}{7}(\cos\gamma - \sin\gamma) + \frac{2}{7} A_\Omega,$$

where $z = \tan\alpha = V_y'/V_x'$ is the tangent of the angle of incidence of the particle on the wall and $A_\Omega = d_p \Omega_z'/(2V_x')$ is a parameter that takes into account angular rotation of the particle before collision.

For a collision with slip, the effective coefficients of momentum regeneration have the form

$$k_t = [(\cos\gamma - z\sin\gamma) - \mu_d(1 + e)\varepsilon_0(\sin\gamma + z\cos\gamma)]\cos\gamma - e(z\cos\gamma + \sin\gamma)\sin\gamma,$$
$$k_n = e\Big(\frac{\sin\gamma}{z} + \cos\gamma\Big)\cos\gamma + \Big[\Big(\frac{\cos\gamma}{z} - \sin\gamma\Big) - \mu_d(1 + e)\varepsilon_0\Big(\frac{\sin\gamma}{z} + \cos\gamma\Big)\Big]\sin\gamma, \tag{2.7}$$
$$k_\Omega = A_\Omega - \frac{5}{2} \mu_d (1 + e)\varepsilon_0(\sin\gamma + z\cos\gamma), \qquad \varepsilon_0 = \operatorname{sign}(\cos\gamma - z\sin\gamma + A_\Omega).$$

Relations (2.6) and (2.7) and the expression for the distribution density of conventional probability of the collision angle (1.3) allow one to determine the probability distribution of the effective coefficients of regeneration of the particle momentum relative to the channel wall

$$\Phi(k, \alpha) = \Big[\int_{-\infty}^{\infty} P(\gamma, \alpha) \varphi'(\gamma)\, d\gamma\Big]^{-1} P(\gamma, \alpha)\varphi'(\gamma). \tag{2.8}$$

Here $k = \varphi(\gamma)$ and $\varphi'(\gamma)$ are the monotonic dependence of the regeneration coefficient on the random angle and its derivative, respectively. The probability density (2.8) is normalized to unity.

3. Calculation Results. To illustrate the model, we use the experimental data obtained for glass balls in a horizontal plane channel with a steel surface [4]. The mean height of random bulges on the surface is 25 μm with standard deviation 5 μm. Figure 2a and b shows the averaged coefficients of momentum regeneration in the longitudinal and normal directions, respectively, versus the angle of incidence of the particles. For a

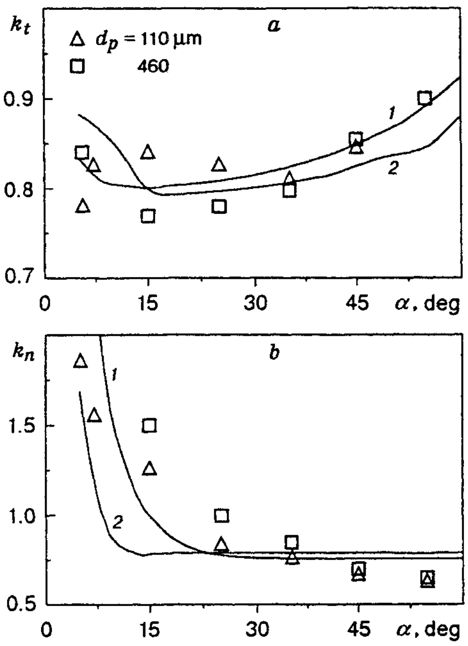

Fig. 2

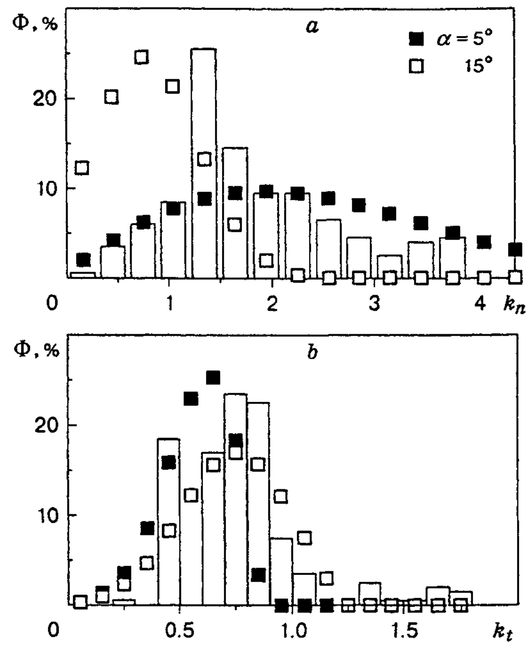

Fig. 3

glass-steel collision, the coefficients of static and dynamic friction and the coefficient of regeneration of the normal component of the momentum are as follows: $\mu_0 = \mu_d = 0.4$ and $e = 0.8$ [9] (triangles and squares in Fig. 2 refer to the experimental data of [4]). Curves 1 and 2 correspond to the particle diameters $d_p = 110$ and 460 μm, respectively. For fine and coarse particles, the variances of the angles of slope of random planes were taken to be $\Delta = 8°$ and 5°, respectively. It is clear that, in contrast to the mean coefficient of momentum regeneration in the longitudinal direction, this coefficient in the normal direction depends considerably on the angle of incidence. Moreover, for angles of incidence $\alpha < 15°$, we have $k_n > 1$. This is associated with the fact that after collision against a random surface, the longitudinal velocity component is converted to the normal component of the reflected particles. The effect of the roughness on the regeneration coefficients becomes weaker as the particle sizes are increased.

In Fig. 3, the predicted density of the probability distribution of the coefficients of momentum regeneration is compared with the experimental data of [4] for various angles of incidence of the particles with $d_p = 110$ μm. Figures 3a and b corresponds to the normal and longitudinal coefficients of momentum regeneration of the particles. The experimental results in the range from 0 to 15° are shown by stepwise functions and the squares refer to calculation results. For small angles of incidence, the surface roughness leads to a wide spectrum of values of the normal and axial velocity components of the reflected particles. It should be noted that the experimental data of [4] on the distribution function for other angles of incidence and particle diameters agree satisfactorily with the calculation results obtained by our model.

Conclusions. Our analysis has shown that, for small angles of incidence of the particles, the coefficient of regeneration of the normal component of the momentum on a rough surface can be greater than unity.

As the angle of incidence increases, the distribution of the probability density becomes narrower and the mean value of the regeneration coefficient becomes smaller than unity. In the case of the longitudinal coefficient of momentum regeneration, the effect of the roughness on the parameters of the reflected particles shows up most noticeably for large angles of incidence of the particles.

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